New Supply Chain Model with Taguchi's Quality Loss and Process Control

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Abstract

In this work, the author addresses an integrated inventory decision model with process control for obtaining the manufacturer's optimal process mean, production cycle time, product warranty, and quality investment and customer's trade credit. Assume that the product characteristic is normally distributed with unknown process mean and known standard deviation. The quality investment can improve the bias and variability of product. Taguchi's asymmetric quadratic quality loss function is applied for evaluating the product quality. A numerical example and sensitivity analysis of some parameters will be provided for illustration in industry application. The management implication of this work is that the manufacturer provides a high quality product/service will satisfy the customer's requirement, promote the customer's expectation, and increase the expected total profit of the supply chain system.

Keywords: Process mean, product warranty, production cycle time, trade credit, quality investment, Taguchi's asymmetric quadratic quality loss function

1. Introduction

The traditional economic manufacturing quantity (EMQ) model assumes that the manufacturing process produces perfect products, and thus it neglects the occurrence of defective items. In EMQ model, one usually considers the minimization of the total expected inventory cost including the set-up cost and holding cost of product per unit time for determining the optimal manufacturing quantity of product. The input resource of production process, e.g., material, operator, machine, tool, method, and environment, usually has the variability between them. Hence, the production process is not always in the state of statistical control. The output product maybe has bias, variability and defective items. The defective products will occur the internal failure cost, e.g., rework and scrap cost, and the external failure cost, e.g., goodwill loss and penalty cost. Previous researchers, such as Porteus (1986) and Rosenblatt and Lee (1986a, 1986b), first incorporated the concept of imperfect quality into the EMQ model. Subsequently, Chung and Hou (2003) and Rahim and Al-Hajailan (2006) presented an imperfect production system with allowable shortage for determining the production run time. Recently, Chen et al. (2015) extended Chung and Hou's (2003) model with the quality loss of conforming items for determining the optimal process mean and production run time. Mandal and Giri (2016) considered the imperfect production system with single-vendor and multi-buyers. The expected demand of each buyer is assumed to be dependent on selling price while lead-time demand of each buyer is assumed to be normally distributed. The vendor also invests money in order to improve the quality of the product and reduce the percentage of defective items.

There are some works that assured a post-sale warranty cost where the nonconforming items are prevalent in the production system. The works of Djamaludin et al. (1994), Yeh and Lo (1998), Yeh, et al. (2000), Wang and Sheu (2000, 2003), Wang (2004, 2006), and Yeh and Chen (2006) can be quoted along this line. Recently, Lin et al. (2017) proposed the integrated production, preventive maintenance, inspection, and inventory model. The imperfect preproduction process considers the minimal repair, preventive maintenance error, and rework. The integrated model needs to obtain the number of inspections, the inspection interval and the economic production quantity. Lin and Gong (2018) considered an integrated manufacturer-buyer supply chain model for vendor's production system with random breakdown. When a breakdown occurs, it needs to perform the correct maintenance to restore the production system. Yu and Chen (2018) considered the single-vendor single-retailer production-inventory model with free replacement warranty policy and quality improvement investment in the defective percentage. They determined the optimal quality improvement investment, number of shipments in a cycle, and the lot size of each shipment under the maximum of the expected annual integrated total profit. Liao

(2018) considered the imperfect production process with maintenance, reliability, and free-repair warranty. The optimal production run time and lot size need to be determined by minimizing the total cost of the deteriorating system. Tai et al. (2019) considered the inventory control problems for deteriorating items with maximum lifetime. Two replenishment policies(quantity-based and time-based policies) and two inspection scenarios (one inspection and continuous monitoring) are addressed. The results indicate that inventory holder adopts the quantity-based policy and performs the continuous monitoring policy for increasing the long-run average profit.

Previous researchers have addressed the permissible delay in payment problem with limited storage capacity, deteriorating production system, ramp-type demand, learning curve production, stock-dependent demand, and two level (supplier-retailer) trade credit period, e.g., Yen et al. (2012), Darzanou and Skouri (2011), Shah and Shah (2012), Teng et al. (2014), Saha (2014), and Chen et al. (2014). Recently, Chuang and Wu (2018) proposed the optimal process mean, quality investment, supplier's number of shipment and retailer's replenishment cycle time settings for the supplier-retailer model with two-level trade credit. Banu and Mondal (2018) proposed an integrated inventory model with/without manufacturer's product warranty. Their model needs to obtain the optimal manufacturer's product warranty, retailer's replenishment cycle time, and customer's trade credit period with the maximum of the expected total profit per unit time. Giri et al. (2018) proposed the lot-for-lot policy of the manufacturer for meeting the demand of the retailer. The manufacturer offers a trade credit to the retailer. The retailer's payment time can be before or after the end of trade credit period and even beyond the cycle time. Giri and Sharma (2019) adopt cash discount and delayed payment while offering trade credit to the retailer. The retailer offers a partial trade credit to his/her customer. This model also compares the different performance between integrated model and non-integrated one. Chang et al. (2019) proposed the EMQ model for perishable goods in supplier-manufacturer-customer chain with advance-cash-credit payment scheme. The manufacturer needs to determine the optimal selling price, production run time, and replenishment cycle time for maximizing the present value of total annual profit by considering the discounted cash flow analysis. Chen and Chou (2021a, 2021b, 20121c, 2021d) have proposed the modified Banu and Mondal's (2018) model without product warranty, with larger-the-better quality characteristic, with joint design of process mean and quality investment, or with the specified process capability index.

In 1986, Taguchi proposed the quadratic quality loss function for measuring the product quality. He redefined the product quality as the loss of society when the product is shipped to the customer. If the product with minimum bias and variability, then it has the optimal target value. Taguchi's (1986) quadratic quality loss function can combine with the on-line and off-line quality control (QC) methods and promote the probability of output product with optimum target value. The optimal process mean setting has been an important topic for modern SPC. By setting the optimal process mean, one can obtain the optimum expected profit/cost per unit product.

The quality investment setting is a long-term method for reducing the bias and variability when the product requires the process improvement. For example, one can adopt the new machine equipment, the new software system, the new material and parts, the new tools for production, the new education training of employee, and the new manufacturing method for improving the production process. Hong et al. (1993), Ganeshan et al. (2001), Chen and Tsou (2003), and Tsou (2006) presented the declining exponential reduction of process mean and standard deviation as the function of quality investment. Recently, Yu and Chen (2018) applied the quality improvement investment policy for addressing the integrated inventory model with product warranty. Chuang and Wu (2019) adopt the quality investment function with declining exponential reduction of process variability for formulating the supply chain model with optimal supplier's process mean and quality investment and retailer's number of shipments, order quantity, and maximal backorder quantity.

For the off-line and on-line QC, we emphasize the product and process optimization for obtaining the minimum expected total loss of society including the producer and the customer. The integrated supply chain model with production, quality, and inventory is an available decision for considering buyer's and seller's common profit. Although the deteriorating production system, product warranty, production cycle time, process mean setting, quality investment for process improvement, and trade credit are different topics for production, quality, and inventory in the supply chain system. If we can integrate these methods for quality/service assurance and improvement, then the business performance will have the significant promotion. The above-mentioned Chuang and Wu's (2018) supplier-retailer model can be extended to the manufacturer-retailer system with optimal product and process parameters. For Chung and Hou's (2003) deteriorating production model, one can consider the optimal quality level and product warranty problems in their model. For Banu and Mondal's (2018) model, one can address the process control of quality in their model. In this work, the author proposes the integration of modified inventory decision model for obtaining the optimal product and process parameters based on the maximum expected total profit including manufacturer and retailer per unit time. In modified model I, one needs to determine the manufacturer's optimal process mean, production cycle time, product warranty, and customer's trade credit period for maximizing the expected total profit including the manufacturer and retailer per unit time. In modified model II, one needs to determine the manufacturer's optimal quality investment, production cycle time, product warranty, and customer's trade credit period for maximizing the expected total profit per unit time. One can expect that the modified model II has the larger expected total profit than that of modified model I.

The main difference between the modified Chung and Hou's (2003) and Banu and Mondal's (2018) models and the original ones is that the former addresses the unknown process mean setting and process variability reduction by quality investment. The long-term quality improvement through quality investment can promote the expected total profit of supply chain system. The modified models are the trade-off problem between the manufacturer and the retailer. Hence, the optimal design of the product and process parameters based on the expected total profit per unit time needs to be considered. The important contribution of this study is that the determination of optimal product and process parameters under the maximum of the expected total profit per unit time by considering buyer and seller. The management implication and theoretical contributions of this work is that the integrated production and inventory model can provide a product with high quality and reliability assurance to the customer and promote the customer's expectation with the maximum expected total profit for the supply chain system. Figure 1 shows that the research structure of this work.

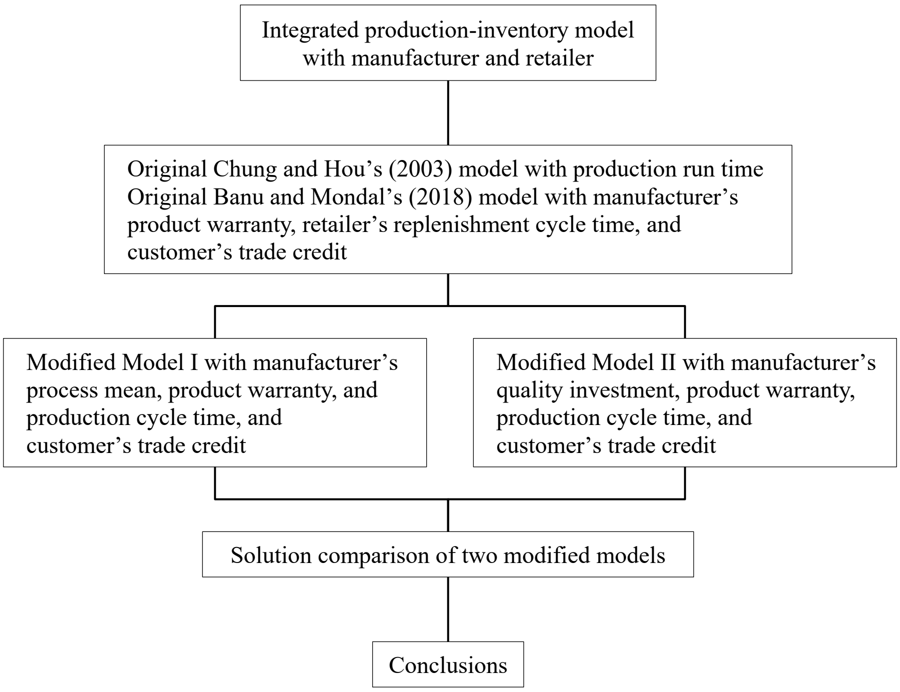

Figure 1. The Research Structure of This Work

2. Assumptions

2.1 Assumptions of Original Models

2.1.1 Assumptions of Original Chung and Hou's (2003) Model

1. The demand rate is constant and deterministic.
2. Shortages are allowed.
3. The elapsed time of production process shifting from an in-control state to an out-of-control is an exponential distribution with parameter λ.
4. The production process is brought back to the in-control state with each setup.

2.1.2 Assumptions of Original Banu and Mondal's (2018) Model

1. The manufacturer produces a single product over infinite time horizon and supplies Q items to the retailer in a lot.
2. The manufacturer provides product warranty period W for the produced products and they are repairable and free repair within the product warranty period.
3. The manufacturer offers warranty dependent trade credit period $M = M_0 - \alpha_0 W$
4. The payment to the manufacturer is carried out at the end of business cycle. So, the retailer has the pay an interest $I_c$to the manufacturer at a

rate of on the remaining amount of stock after the trade credit period M.

5. To increase the demand of the product, the retailer has offered a trade credit period N to all customers. Each customer must pay his/her dues before M.
6. The capital opportunity cost is considered only for the manufacturer within the time gap between product shipped and paid.
7. The demand function has been considered as a linear combination of W and N. At the time of payment of customer, the retailer will face with an extra cost to collect the money. It is considered as fixed cost F.
8. The retailer offers the facility of trade credit period only during the period [0, M-N] and after that no trade credit period be given to the customer. Thus, the demand function linearly depends on W and N during the period [0, M-N] and then it decreases with time.

2.2 Assumptions of Integration of Modified Chung and Hou's (2003) Model and Banu and Mondal's (2018) Model

Most of the assumptions in the integration of modified Chung and Hou's (2003) model and Banu and Mondal's (2018) model are the same as those of the original ones. However, the different assumptions are as follows:

1. The quality characteristic is normally distributed with unknown process mean and known process standard deviation.
2. Taguchi's quadratic quality loss function is applied for measuring the product quality.
3. The quality characteristic of product is above the upper specification limit will be scrapped. The quality characteristic of product is below the lower specification limit will be reworked.
4. The rework item becomes a conforming product.
5. The failure time of conforming item obeys the Weibull distribution with parameters θ_1 and ρ_1 with expected value of time $\theta_1\Gamma(1+1/\rho_1)$.
6. The failure time of rework item obeys the Weibull distribution with parameters θ_2 and ρ_2 with expected value of time $\theta_2\Gamma(1+1/\rho_2)$.
7. The manufacturer produces a single product during the production cycle time T which is the retailer's replenishment cycle time.
8. The quality investment function is defined as the declining exponential reduction of process mean and process standard deviation.

3. Integration of Modified Chung and Hou's and Banu and Mondal's Model

3.1 Modified Model I with Process Mean Setting

Similar to Chung and Hou (2003) and Banu and Mondal (2018), the modified model I has the following formulates for the manufacturer's and retailer's revenue and cost:

3.1.1 Manufacturer's Revenue and Cost

(1) The average sales revenue per unit time is given by $\frac{UQ}{T}$.

(2) The average material cost per unit time is given by $\frac{c\mu_y Q}{T}$.

(3) The setup cost per unit time is given by $\frac{K}{T}$.

(4) The holding cost per unit time is given by $(h+\pi)\frac{(p-D)}{2t}T_1^2$, where $T_1=\frac{\pi}{p-D}$ and $t=\frac{Q}{p}$.

(5) The backorder cost per unit time is given by $h[\frac{(p-D)t}{2}-(p-D)T_1]$, where $T_1=\frac{\pi}{p-D}$ and $t=\frac{Q}{p}$.

(6) The average rework cost of product per unit time is given by $\frac{C_R p[1-\Phi(\frac{USL-\mu_y}{\sigma_y})]\{t+\frac{[exp(-\lambda t)-1]}{\lambda}\}}{T}$, where $t=\frac{Q}{p}$.

(7) The average scrap cost of product per unit time is given by $\frac{C_j p\Phi(\frac{LSL-\mu_y}{\sigma_y})\{t+\frac{[exp(-\lambda t)-1]}{\lambda}\}}{T}$, where $t=\frac{Q}{p}$.

(8) The average quality loss of conforming product per unit time is given by $\frac{[pt-E(Z)]E[Loss(Y)]}{T}$, where $E(Z)=p[\Phi(\frac{LSL-\mu_y}{\sigma_y})+1-\Phi(\frac{USL-\mu_y}{\sigma_y})]\{t+\frac{[exp(-\lambda t)-1]}{\lambda}\}$, $t=\frac{Q}{p}$ and $E[Loss(Y)]=\int_{LSL}^{y_0}k_1(y-y_0)^2f(y)dy+\int_{y_0}^{USL}k_2(y-y_0)^2f(y)dy$

(9) The average opportunity cost per unit time is given by $\frac{UI_pQM}{T}$.

(10) The average product warranty service cost per unit time is given by $\frac{f\{[\Phi(\frac{USL-\mu_y}{\sigma_y})-\Phi(\frac{LSL-\mu_y}{\sigma_y})]\int_0^W h_1(x_1)dx_1+[1-\Phi(\frac{USL-\mu_y}{\sigma_y})]\int_0^W h_2(x_2)dx_2\}Q}{T}$

Hence, the average total profit for the manufacturer includes average sales revenue – average material cost – average setup cost – average holding cost – average backorder cost – average rework cost – average scrap cost – average quality loss of conforming product – average opportunity cost – average product warranty service cost, that is,

$$ETP_M=\frac{UQ}{T}-\frac{C\mu_yQ}{T}-\frac{K}{T}-(h+\pi)\frac{(p-D)}{2t}T_1^2-h\left[\frac{(p-D)t}{2}-(p-D)T_1\right]-\frac{C_Rp\left[1-\Phi\left(\frac{USL-\mu_y}{\sigma_y}\right)\right]\left\{t+\frac{[exp(\lambda t)-1]}{\lambda}\right\}}{T}-\frac{C_jp\Phi\left(\frac{LSL-\mu_y}{\sigma_y}\right)\left\{t+\frac{[exp(-\lambda t)-1]}{\lambda}\right\}}{T}-\frac{[pt-E(Z)]E[Loss(Y)]}{T}-\frac{UI_pQM}{T}-\frac{f\{[\Phi(\frac{USL-\mu_y}{\sigma_y})-\Phi(\frac{LSL-\mu_y}{\sigma_y})]\int_0^W h_1(x_1)dx_1+[1-\Phi(\frac{USL-\mu_y}{\sigma_y})]\int_0^W h_2(x_2)dx_2\}Q}{T} \quad (1)$$

3.1.2 Retailer's Revenue and Cost

(1) The average sales revenue per unit time is given by $\frac{sQ}{T}$.

(2) The average interest earned per unit time is given by $\frac{sI_eD}{2T}[T^2-2N(M-N)-\frac{\beta_3}{3}(T-M+N)^3]$, where $D=D_0(1+\beta_1W+\beta_2N)$,

$Q = D[T - \frac{\beta_3}{2}(T - M + N)^2]$ and $M = M_0 - \alpha_0 W$.

(3) The average purchase cost per unit time is given by $\frac{UQ}{T}$.

(4) The average holding cost is given by $\frac{h_s D}{2T}[T^2 - \frac{\beta_3}{3}(T - M + N)^2(2T + M - N)]$, where $D = D_0(1 + \beta_1 W + \beta_2 N)$ and $M = M_0 - \alpha_0 W$.

(5) The average ordering cost per unit time is given by $\frac{A_s}{T}$.

(6) The average interest charged per unit time is given by $\frac{UI_c D(T-M)^2}{2T}\{1 - \beta_3[\frac{2}{3}(T - M) + N]\}, D = D_0(1 + \beta_1 W + \beta_2 N)$ and $M = M_0 - \alpha_0 W$.

(7) The average collection cost per unit time is given by $\frac{F}{T}$.

Hence, the average total profit for the retailer includes average sales revenue + average interest earned – average purchase cost – average holding cost – average ordering cost – average interest charged – average collection cost, that is,

$$ETP_R = \frac{sQ}{T} + \frac{sI_e D}{2T}\left[T^2 - 2N(M - N) - \frac{\beta_3}{3}(T - M + N)^3\right] - \frac{UQ}{T} - \frac{h_s D}{2T}\left[T^2 - \frac{\beta_3}{3}(T - M + N)^2(2T + M - N)\right] - \frac{A_s}{T} - \frac{UI_c D(T-M)^2}{2T}\left\{1 - \beta_3\left[\frac{2}{3}(T - M) + N\right]\right\} - \frac{F}{T} \quad (2)$$

Therefore, the average total profit including the manufacturer and the retailer per unit time is as follows:

$$ETP(\mu_y, N, T, W) = ETP_M + ETP_R \quad (3)$$

where μ_y is the manufacturer's process mean; *N* is the customer's trade credit period offered by the retailer (year); *T* is the manufacturer's production cycle time (year); *W* is the warranty period of the product offered by the manufacturer (year).

3.1.3 Solution Procedure

One cannot prove that that the Hessian matrix of Eq. (3) is a negative-defined matrix with respect to (μ_y, N, T, W) because of the cumulative distribution function of the standard normal random variable$\Phi(\cdot)$. Hence, we do not have the closed-form solution for (μ_y, N, T, W). The optimal solution of Eq. (3) is found numerically. The solution procedure of Eq. (3) is as follows:

Step 1. Set the maximum $N = M_0$, maximum $W = \theta_2 \Gamma(1 + 1/\rho_2)$, and maximum $T = 1$.

Step 2. For the given *N*, *T*, and *W*, one can adopt the direct search method for obtaining the optimal μ_y between *LSL* and *USL* with maximum expected total profit per unit time for Eq. (3).

Step 3. Repeat step 2 until $N = M_0$, $W = \theta_2 \Gamma(1 + 1/\rho_2)$, and $T = 1$. The combination of (μ_y^*, N^*, T^*, W^*) with maximum $ETP(\mu_y, N, T, W)$ is the optimal solution.

3.2 Modified Model II with Quality Investment Setting

Consider the quality investment can decrease the process bias and variability. Denote Inv be the quality investment and $f(y, Inv)$ be the probability density function of quality characteristic *Y* with quality investment, where $f(y, Inv) = \frac{1}{\sqrt{2\pi}\sigma_I} exp\left(-\frac{1}{2}\left(\frac{y-\mu_I}{\sigma_I}\right)^2\right), -\infty < y < \infty$; $\mu_I^2 = y_0^2 + (\mu_y^2 - y_0^2)\exp(-\beta Inv), \beta > 0$; $\sigma_1^2 = \sigma_t^2 + (\sigma_y^2 - \sigma_t^2)exp(-\alpha Inv), \alpha > 0$; μ_I is the process mean associated with quality investment, Inv; σ_I is the process standard deviation associated with quality investment, Inv; μ_y is the optimal setting value of the process mean from modified model I; y_0 is the target value of the process mean; σ_y is the known process standard deviation; σ_t is the minimum achievable level of the process standard deviation; α is the quality investment function parameter for process standard deviation; β is the quality investment function parameter for process mean.

Similar to the above-mentioned Eq. (3), the average total profit including the manufacturer and the retailer per unit time is

$$ETP(Inv, N, T, W) = ETP_M - \frac{Inv}{T} + ETP_R \quad (4)$$

where *Inv* is the manufacturer's quality investment; *N* is the customer's trade credit period offered by the retailer (year); *T* is the manufacturer's production cycle time (year); *W* is the warranty period of the product offered by the manufacturer (year).

3.2.1 Solution Procedure

One cannot prove that that the Hessian matrix of Eq. (4) is a negative-defined matrix with respect to (Inv, N, T, W) because of the cumulative distribution function of the standard normal random variable$\Phi(\cdot)$. Hence, we do not have the closed-form solution for (Inv, N, T, W). The optimal solution for Eq. (4) is found numerically. The solution procedure for Eq. (4) is as follows:

Step 1. Set the maximum $N = M_0$, maximum $W = \theta_2 \Gamma(1 + 1/\rho_2)$, and maximum$T = 1$.

Step 2. For the given *N*, *T*, and *W*, one can adopt the direct search method for obtaining the optimal Inv with maximum expected total profit per unit time for Eq. (4).

Step 3. Repeat step 2 until $N = M_0$, $W = \theta_2 \Gamma(1 + 1/\rho_2)$, and $T = 1$. The combination of (Inv^*, N^*, T^*, W^*) with maximum $ETP(Inv, N, T, W)$ is the optimal solution.

4. Numerical Example and Sensitivity Analysis

Consider a product with the quality characteristic, Y, is normally distributed with adjustable process mean μ_y and known process standard deviation $\sigma_y = 0.25$. The lower specification limit of product is LSL = 3 and the upper specification limit of product is USL = 7. The target value of product is $y_0 = 4.5$. The production rate per month is p = 200 items. The base demand rate per month is $D_0 = 100$ items without product warranty and trade credit periods. The setup cost for each production run is K = 5 The holding cost per items per month is h = 4. The elapsed time of process shifting from an in-control state to an out-of-control state is an exponential distribution with parameter $\lambda = 0.05$. The rework cost per item is $C_R = 4$ when the quality characteristic is above the USL. The scrap cost per item is $C_j = 3$ when the quality characteristic is above the LSL. The quality loss coefficient is $k_1 = 5$ when LSL <Y <y_0 and the quality loss coefficient is $k_2 = 10$ when y_0<Y<USL. The backorder cost per item per month is $\pi = 2$. The manufacturer provides the product warranty period W and the trade credit period M to the retailer. Hence, the manufacturer has the capital opportunity cost within the time gap between product shipped and paid. Other parameters for manufacturer's revenue and cost are as follows: U=40, c=3, $I_p = 0.08$, $\theta_1 = 1$, $\rho_1 = 0.5$, $\theta_2 = 0.9$, $\rho_2 = 0.5$, and $f = 30$.

There are sales revenue, purchase cost, holding cost, and ordering cost of product for the retailer. The retailer also provides the trade credit period N which is less than or equal to M to the customer for promoting the demand of customers. Hence, the retailer has the interest earned, interest charged, and collection cost of capital. some parameters for retailer's revenue and cost are as follows: s=70, $I_e = 0.08$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.4$, $\alpha_0 = 0.1$, $M_0 = 0.8$, $h_s = 2$, $A_S = 30$, $I_c = 0.12$, and$F = 30$.

The above manufacturer-retailer (production-inventory) model I hopes to obtain the manufacturer's optimal process mean(μ_y), production cycle time (T), product warranty (W) and customer's trade credit period offered by retailer (N) under the maximum expected total profit including the manufacturer and retailer per month. The manufacturer also hopes to have the quality improvement of product by the activity of quality investment for increasing the expected total profit including the manufacturer and retailer per month. Hence, the modified model II hopes to obtain the manufacturer's optimal quality investment (Inv), production cycle time (T), component warranty (W) and customer's trade credit period offered by retailer (N) under the maximum expected total profit including the manufacturer and retailer per month.

By solving Eq. (3), we have the optimal $\mu_y^* = 4.19, N^* = 0.7, T^* = 0.5$, and $W^* = 0$ with corresponding $ETP_M = 2837.31, ETP_R = 3441.51$, and $ETP(\mu_y, N, T, W) = 6278.82$.

Let $\alpha = 0.05, \beta = 0.01$, and $\sigma_t = 0.2$. By solving Eq. (4), we have the optimal $Inv^* = 5, \mu_y^* = 4.235, \sigma_y^* = 0.225, N^* = 0.7, T^* = 0.5$, and $W^* = 0$ with corresponding $ETP_M = 2838.87, ETP_R = 3441.51$, and $ETP(Inv, N, T, W) = 6280.38$.

Tables 1-2 shows the sensitivity analysis of some parameters for modified models I and II. From Tables 1-2, we have some conclusions:

1. The sale price per item for the manufacturer has a major effect on the expected total profit of manufacturer, the expected total profit of retailer, and the expected total profit of supply chain system.
2. The material cost per item for the manufacturer has a major effect on the expected total profit of manufacturer and the expected total profit of supply chain system.
3. The base demand rate per unit time when there is no effect of product warranty and trade credit has a major effect on the expected total profit of manufacturer, the expected total profit of retailer, and the expected total profit of supply chain system.
4. The retailer's selling price per item has a major effect on the expected total profit of retailer and the expected total profit of supply chain system.
5. The effective parameter for trade credit period has a major effect on the expected total profit of manufacturer, the expected total profit of retailer, and the expected total profit of supply chain system.
6. The decreasing parameter for demand has a major effect on the expected total profit of retailer and the expected total profit of supply chain system.
7. The modified model II has almost larger expected total profit of supply chain system than that of modified model I.

Table 1: The Sensitivity Analysis of Some Parameters for Modified Model I

U	μ_y^*	N^*	T^*	W^*	ETP_M	ETP_R	$ETP(\mu_y, N, T, W)$
32	4.19	0.7	0.5	0	**1940.18**	**4408.82**	**6349.00**
48	4.19	0.7	0.5	0	**3734.43**	**2474.20**	**6208.63**
c	μ_y^*	N^*	T^*	W^*	ETP_M	ETP_R	$ETP(\mu_y, N, T, W)$
2.4	4.24	0.7	0.5	0	**3140.24**	3441.51	**6581.75**
3.6	4.13	0.7	0.5	0	**2538.28**	3441.51	**5979.78**

D_0	μ_y^*	N^*	T^*	W^*	ETP_M	ETP_R	$ETP(\mu_y, N, T, W)$
80	4.19	0.7	0.5	0	**2257.11**	**2717.21**	**4974.32**
120	4.19	0.7	0.5	0	**3423.52**	**4165.81**	**7589.33**
s	μ_y^*	N^*	T^*	W^*	ETP_M	ETP_R	$ETP(\mu_y, N, T, W)$
56	4.19	0.7	0.5	0	2837.31	**1749.65**	**4586.96**
84	4.19	0.7	0.5	0	2837.31	**5133.37**	**7970.67**
β_2	μ_y^*	N^*	T^*	W^*	ETP_M	ETP_R	$ETP(\mu_y, N, T, W)$
0.32	4.19	0.7	0.5	0	**2709.87**	**3283.07**	**5992.94**
0.48	4.19	0.7	0.5	0	**2965.03**	**3599.95**	**6564.98**
β_3	μ_y^*	N^*	T^*	W^*	ETP_M	ETP_R	$ETP(\mu_y, N, T, W)$
0.32	4.19	0.7	0.5	0	2876.15	**3489.58**	**6365.73**
0.48	4.19	0.7	0.4	0	2873.73	**3323.33**	**6197.07**

Table 2: The Sensitivity Analysis of Some Parameters for Modified Model II

U	Inv^*	N^*	T^*	W^*	ETP_M	ETP_R	$ETP(Inv, N, T, W)$
32	5	0.7	0.5	0	**1941.75**	**4408.82**	**6350.57**
48	5	0.7	0.5	0	**3735.99**	**2474.20**	**6210.19**
c	Inv^*	N^*	T^*	W^*	ETP_M	ETP_R	$ETP(Inv, N, T, W)$
2.4	7	0.7	0.5	0	**3144.49**	3441.51	**6586.00**
3.6	0	0.7	0.5	0	**2536.11**	3441.51	**5977.62**
D_0	Inv^*	N^*	T^*	W^*	ETP_M	ETP_R	$ETP(Inv, N, T, W)$
80	5	0.7	0.5	0	**2257.36**	**2717.21**	**4974.57**
120	5	0.7	0.5	0	**3426.39**	**4165.81**	**7592.20**
s	Inv^*	N^*	T^*	W^*	ETP_M	ETP_R	$ETP(Inv, N, T, W)$
56	5	0.7	0.5	0	2838.87	**1749.65**	**4588.52**
84	5	0.7	0.5	0	2838.87	**5133.37**	**7972.24**
β_2	Inv^*	N^*	T^*	W^*	ETP_M	ETP_R	$ETP(Inv, N, T, W)$
0.32	5	0.7	0.5	0	**2711.15**	**3283.07**	**5994.21**
0.48	5	0.7	0.5	0	**2966.88**	**3599.95**	**6566.83**
β_3	Inv^*	N^*	T^*	W^*	ETP_M	ETP_R	$ETP(Inv, N, T, W)$
0.32	5	0.7	0.5	0	2877.80	**3489.58**	**6367.38**
0.48	5	0.7	0.4	0	2875.37	**3323.33**	**6198.70**

5. Conclusions

In this paper, the author presents the modified Chung and Hou (2003) and Banu and Mondal's (2018) models for determining the optimal process mean/ quality investment, production cycle time, product warranty, and customer's trade credit. From the above numerical results, the modified model with quality investment has larger average total profit including the manufacturer and the retailer per unit time. Hence, the quality investment is a effective tool for promoting the expected total profit of the supply chain system. The optimal process mean setting should be firstly considered in the manufacturer's process. Then the quality investment should be executed for pursuing the target value of product. The integrated application of this work is available for production and operational management in the production quantity, inventory management, process control, process improvement, product reliability, and quality assurance of product to the customer. By considering this integrated production-inventory model, the satisfaction level of customer will have the significant promotion for the buyer-seller system. Further study should consider modified model with multiple quality characteristics, multi-suppliers, and multi-retailers for determining the optimal process and product parameters.

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Appendix: Notations for Mathematical Models

U	the sale price per item for the manufacturer (= the retailer's purchase price per item)
c	the material cost per item for the manufacturer
f	the warranty service cost per item
σ_y	the process standard deviation
μ_y	the process mean
Q	the manufacturer's production quantity per production cycle time (= the retailer's order quantity per replenishment cycle time)
T	the manufacturer's production cycle time (= the retailer's replenishment cycle time)
K	the setup cost for the manufacturer
h	the holding cost per item for the manufacturer
π	the backorder cost per item for the manufacturer
p	the production rate per unit time
D	the demand rate with product warranty and trade credit per unit time, D < p
D_0	the base demand rate per unit time when there is no effect of product warranty and trade credit
t	the production run time in a production cycle time (= Q/ p)
T_1	the production time when backorder is replenished (=$\pi/(p-D)$)
C_R	the rework cost per item for the manufacturer
C_j	the scrap cost per item for the manufacturer
λ	the parameter of the exponential distribution for the production process shifting to the out-of-control state
LSL	the lower specification limit of product
USL	the upper specification limit of product
E(Z)	the expected number of defective items in a production cycle time
$\varphi(\cdot)$	the probability distribution function of standard normal random variable
$\Phi(\cdot)$	the cumulative distribution function of standard normal random variable
E[Loss(Y)]	the expected quality loss per item
Y	the normal quality characteristic of product
y_0	the target value of product
k_1	the quality loss coefficient when LSL <Y <y_0
k_2	the quality loss coefficient when y_0<Y<USL
$h_1(x_1)$	the Weibull failure rate function of conforming item, $h_1(x_1)=\frac{\rho_1}{\theta_1}(\frac{x_1}{\theta_1})^{\rho_1-1}exp[-(\frac{x_1}{\theta_1})^{\rho_1}]$ with expected value $E(X_1)=\theta_1\Gamma(1+1/\rho_1)$, where $\Gamma(\cdot)$is the gamma function
$h_2(x_2)$	the Weibull failure rate function of rework item, $h_2(x_2)=\frac{\rho_2}{\theta_2}(\frac{x_2}{\theta_2})^{\rho_2-1}exp[-(\frac{x_2}{\theta_2})^{\rho_2}]$ with expected value $E(X_2)=\theta_2\Gamma(1+1/\rho_2)$ and $E(X_2)\le E(X_1)$, where $\Gamma(\cdot)$is the gamma function
M	the retailer's trade credit period offered by manufacturer with product warranty period (year),$M=M_0-\alpha_0 W$
M_0	the fixed trade credit period offered by the manufacturer when there is no product warranty period
α_0	the effective parameter associated to the product warranty period,$0<\alpha_0<1$
N	the customer's credit period offered by the retailer (year), $N\le M$
W	the warranty period of the product offered by the manufacturer (year)
β_1	the effective parameter for product warranty period, $\beta_1>0$
β_2	the effective parameter for trade credit period, $\beta_2>0$
β_3	the decreasing parameter for demand, $\beta_3>0$
s	the retailer's selling price per item, s > U
F	the collection cost per item for the retailer
h_s	the retailer's holding price per item
I_e	the rate of interest earned of retailer
I_c	the rate of interest charged for the remaining stock from M to T to manufacturer, after offered trade credit period
I_p	the rate of interest for calculating manufacturer's opportunity interest loss due to delay payment
A_s	the ordering cost for the retailer

$D=D_0(1+\beta_1 W+\beta_2 N)$

$Q=D(1+\beta_1 W+\beta_2 N)[T-\frac{\beta_3}{2}(T-M+N)^2]$.