The Joint Design of Specification Limits and Quality Investment

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Received 2 November 2022; received in revised form 21 December 2022; accepted 27 December 2022

Abstract

Statistical quality control is generally considered a useful methodology in the process of continuous quality improvement, in which product inspection, process control and quality design are three important aspects. In 1996, Pulak and Al-Sultan proposed the optimum process mean selection under the single sampling rectifying inspection plan. Since then, the three aspects in statistical quality control have been integrated in many research works. In the present paper, two modified Pulak and Al-Sultan's models are developed by incorporating the specification limits of process characteristic and quality investment with the specified value of the process capability index C_{pm} or C_{pmk}, where the optimal quality investment and the specification limits of process characteristic are determined based on maximization of the expected profit per item. The direct search method is applied to determine the optimal solution for the two modified models. Two numerical examples are given and the sensitivity analyses are conducted to investigate the effects of model parameters on the optimal solution. According to the study, it may be concluded that the target value of process mean has a major effect on quality investment, which indicates that the quantity of quality investment should seriously depend upon the condition of the mean of process characteristic. Meanwhile, if the process improvement includes quality investment, the improved process mean and standard deviation are able to attain the specified value of process capability index; however, if quality investment is not involved in the process improvement, the value of process capability index needs to be satisfied for assuring the output product quality by setting the symmetric specification limit coefficient.

Keywords: Quality investment, process capability value, specification limits, single sampling rectifying inspection plan

1. Introduction

Continuous quality improvement is always the way for enterprises to achieve customers' satisfaction and to enhance their competitiveness. Statistical quality control (SQC) is generally considered a useful approach in the process of continuous quality improvement, where some predictive methods, such as the on-line quality control and off-line quality control techniques, are applied. Consequently, product inspection, process control and quality design are three important topics in SQC. In the area of product inspection, acceptance sampling plan is often used in the product output stage to determine the quality of lot. In the area of process control, control chart technique and process capability index are usually adopted for controlling the quality of a production process. Since quality improvement is always the long-term objective of quality management policy for enterprises, in the area of quality design, the design of experiments and/or Taguchi's method are helpful for selecting the appropriate level combination of production factors in order to reduce the bias and variability of products. In the present paper, the acceptance sampling inspection plan and statistical process control (SPC) are integrated for implementing continuous quality improvement for enterprises.

The process characteristic (product characteristic) is often used in the manufacturing industry, e.g., semiconductor, integrated circuit, and automation industry. The process characteristic of product is usually considered as the normal distribution in quantitative quality control.

The on-line 100% inspection or the sampling rectifying inspection can be used as a short-term method for controlling the quality of products which are shipped to customers. The optimal process mean setting is a cared aspect for SPC because the process mean always affects the expected total profit/cost per unit. In 1996, Pulak and Al-Sultan (1996) presented a single sampling rectifying inspection plan to determine the optimal process mean with maximization of the expected profit per item, in which the process characteristic is a larger-the-better characteristic and is assumed to be normally distributed with known process standard deviation. Recently, Wu and Liu (2014), Liu et al. (2014) and Liu and Wu (2014) also have presented the integration model of acceptance sampling plan and process capability index for controlling the lot fraction of defectives.

The process capability index is generally applied to examine whether or not a production process is capable. The product and process optimization may be achieved by minimizing the expected total loss of society with optimally determined product/process parameters. Boyles (1991) indicated that the process capability index C_{pm} is the same as the indicator proposed by Taguchi (1986). Pearn, et al. (1992) introduced the process capability index C_{pmk} by considering the difference between the mean and specification center of the process characteristic. Modern manufacturing processes often require the very low parts per million (PPM) fraction of defectives. By specifying the value of capability index, e.g., C_{pm} or C_{pmk}, for the production process, the output quality of products and the constrained loss of customers may be guaranteed.

The quality investment is an alternative way for continuous quality improvement in the long term. For example, the enterprise may adopt the new machine equipment, the new software system, or the new manufacturing techniques to improve the performance of production processes. The quality investment is generally expressed as the declining exponential reduction function of the process mean and standard deviation, e.g., see Hong et al. (1993), Ganeshan et al. (2001), Chen and Tsou (2003), and Tsou (2006). Furthermore, Abdul-Kader et al. (2010) employed quality investment function given in Chen and Tsou (2003) to determine the optimum quality investment and corresponding improved process mean and standard deviation. Recently, Yu and Chen (2018) applied the quality improvement investment policy for addressing the integrated inventory model with product warranty. Chuang and Wu (2018) proposed the optimal process mean, quality investment, supplier's number of shipment and retailer's replenishment cycle time settings for the supplier-retailer model with two-level trade credit. Chuang and Wu (2019) adopted the quality investment function with declining exponential reduction of process variability for formulating the supply chain model with optimal supplier's process mean and quality investment and retailer's number of shipments, order quantity, and maximal backorder quantity. Chen and Chou (2020a, 2020b) proposed the integrated models with application policy of specification limits and quality investment for rectifying inspection plan and 100% inspection.

Although the sampling inspection plan, determination of specification limits of process characteristic, process capability index and quality investment are considered different quality tools for continuous quality improvement, these tools may be integrated for quality assurance, such that the quality performance of the products could be significantly promoted. Pulak and Al-Sultan's (1996) model with constant standard deviation did not consider the product quality to the customer. Thus, the process improvement and the constrained loss of product to the customer should be included in their model. In the present paper, the model in Pulak and Al-Sultan (1996) are modified with specified value of process capability index to determine the optimal specification limits of process characteristic and quality investment. Two modified models are respectively developed based on the specified values of C_{pm} and C_{pmk}. The two modified models combining with process parameters and quality investment policy would be able to enhance the benefits of manufacturer and customers. In the next section, the model given in Pulak and Al-Sultan (1996) is briefly reviewed. Then, the optimization mathematical models and their solution procedures are presented. Finally, the numerical example and sensitivity analysis of model parameters are provided for illustration.

2. Review of Pulak and Al-Sultan's Model

Assume that the process characteristic is normally distributed with known standard deviation (σ_0) and lower specification limit (L). A rectifying inspection plan is executed to decide the quality of the lot. Specifically, a sample size of n is drawn from the lot of size N and then the sample is inspected. Based on the number of defective units in the sample with size n, the decision is made as follows: if the number of defective units in the sample is less than or equal to d_0, then the product lot is accepted and is sold per unit at a price A_2; however, if the number of defective units in the sample is greater than d_0, the product lot is rejected, then is replaced by conforming ones, and is sold per unit at a price A_1. Pulak and Al-Sultan (1996) developed a model to determine the optimum process mean based on maximization of the expected profit per item. The objective function of the model in Pulak and Al-Sultan (1996, pp. 732-733) is

$$ETP(\mu_0) = [A_1 - \frac{R_L}{N} - (1 - \frac{n}{N})I_c]P(D > d_0) + A_2 P(D \leq d_0) - c\mu_0 - \frac{n}{N}I_c \quad (1)$$

where $ETP(\mu_0)$ is the expected profit function per item; A_1 is the selling price per item for items in 100% inspected lot; A_2 is the selling price per item for items in the lot accepted by acceptance sampling, and $A_2 \leq A_1$; n is the sample size; D is the number of non-conforming items found in the sample size n; d_0 is the allowance number of non-conforming items found in a sample size n; $P(D \leq d_0)$ is the probability of accepting the lot ($= 1 - P(D > d_0)$); c is the processing cost per item; μ_0 is the process mean; N is the lot size; I_c is the inspection cost per item; R_L is the expected cost of replacing all rejected items found in a rejected lot ($= R_I \cdot d_{rl}$); R_I is the cost of replacing a defective item by an acceptable item; d_{rl} is the expected number of defective items in a rejected lot, which is equal to the expected number of defectives found in the sample, given that the lot was rejected, plus the expected number of defectives in the non-sample portion of the lot, i.e., $d_{rl} =$

$E(D|D > d_0) + (N - n)p$; p is the probability of producing a defective item (=$\Phi(\frac{L-\mu_0}{\sigma_0})$); L is the lower specification limit of process characteristic; σ_0 is the process standard deviation; $\Phi(\cdot)$is the cumulative distribution function of the standard normal random variable;

$$E(D|D > d_0) = \frac{\sum_{d=d_0+1}^{n} d\frac{n!}{d!(n-d)!}p^d(1-p)^{n-d}}{1-\sum_{d=0}^{d_0}\frac{n!}{d!(n-d)!}p^d(1-p)^{n-d}} \quad (2)$$

Pulak and Al-Sultan (1996) applied the one-dimensional golden section search method to determine the optimal value of μ_0 for Eq. (1). A sensitivity analysis of the model parameters are also provided in their work. However, their model only considered the process mean setting and neglected the importance of process improvement for quality assurance. Hence, following two modified models will address the constrained loss of product to the customer by quality investment and setting of specified process capability index value.

3. Two Modified Pulak and Al-Sultan's Models with Specified Value of Process Capability Index

The modified model with specified value of C_{pm} addresses the symmetric specification limit coefficient, and the modified model with specified value of C_{pmk} considers the asymmetric specification limit coefficient. Pearn et al. (2006) pointed out that "In general, asymmetric tolerances simply reflect that deviations from the target value are less tolerable in one direction than in the other direction. Asymmetric tolerances can arise from a situation in which the tolerances are symmetric to begin with, but the process follows a non-normal distribution and the data are transformed to achieve approximate normality".

3.1 Assumptions

Some assumptions are the same as the model developed by Pulak and Al-Sultan (1996) except for

1. The process characteristic has both-sided specification limits.
2. The function of quality investment is the declining exponential reduction function of process mean and standard deviaiton.
3. The target value of process characteristic is equal to the center point of specification limits.
4. The probability of accepting a lot is computed by using Poisson distribution.
5. The process characteristic is normally distributed with known process mean and standard deviation.

The above-mentioned assumptions 1, 2, and 5 are based on the process mean setting model of Pulak and Sultan's (1996) obtaining the initial process mean. Then, the process parameters can been improved by quality investment. To simply the computation, one considers assumptions 3 and 4.

3.2 Modified Pulak and Al-Sultan's Models with Quality Investment and Specified Process Capability C_{pm} Value

Chen and Tsou (2003) pointed out that "One can use the regression analysis model for the checking the goodness of fitting the declining exponential function of the capital investment and estimating the parameters of declining exponential function". The unit of quality investment is the money amount spent for improving the product quality. According to Chen and Tsou (2003), both the process mean and standard deviation may be expressed as a declining exponential reduction function of quality investment. That is, from Chen and Tsou (2003), the improved process mean μ_y and the improved process standard deviation σ_y can be mathematically expressed by

$$\mu_y^2 = \mu_T^2 + (\mu_0^2 - \mu_T^2)\exp(-\beta INV) \quad (3)$$

$$\sigma_y^2 = \sigma_T^2 + (\sigma_0^2 - \sigma_T^2)\exp(-\alpha INV) \quad (4)$$

where μ_0 is the known process mean from the solution of the original model; $\sigma_0$is the known process standard deviation; $\mu_T$is the target value of process mean; σ_T is the target value of process standard deviation; α is the function parameter for the process mean; β is the function parameter for the process standard deviation; *INV* is the quality investment.

Let *L* and *U* denote the lower and upper specification limits of process characteristic, respectively. Let *a* be the specification coefficient, that is, $U = \mu_y + a\sigma_y$ and $L = \mu_y - a\sigma_y$, where $a > 0$. Assume the target value of process characteristic, *T*, is equal to the specification center, i.e., $T = \frac{L+U}{2}$. Based on the definition of C_{pm}, it may be shown that

$$C_{pm} = \frac{U-L}{6\sqrt{\sigma_y^2+(\mu_y-T)^2}} = \frac{2a\sigma_y}{6\sigma_y} = \frac{a}{3} \quad (5)$$

From statistical theory, the probability of binomial distribution can be approximated by the Poisson distribution under the conditions that the sample size, *n*, is sufficiently large and the probability of having a defective item, *p*, approaches zero. Therefore, if the specified value of the process capability index C_{pm} or C_{pmk} is large enough, the Poisson distribution may be used to obtain the approximate probability of binomial distribution because the probability of having a defective product is usually very small. Hence, the expected number of defectives found in the sample, given that the lot was rejected, can be obtained as follows:

$$E(D|D > d_0) \approx \frac{np[1-\sum_{d=0}^{d_0-1}\frac{e^{-np}(np)^d}{d!}]}{1-\sum_{d=0}^{d_0}\frac{e^{-np}(np)^d}{d!}} \quad (6)$$

Consequently, the modified Pulak and Al-Sultan's model incorporating the quality investment and the value of C_{pm} can be formulated as follows:

Maximize

$$ETP(INV, C_{pm}) = [A_1 - \frac{R_L}{N} - I_c - c\mu_y] + [A_2 - A_1 + \frac{R_L}{N} + (1 - \frac{n}{N})I_c]P(D \le d_0) - \frac{INV}{N} \quad (7)$$

where

$$p = 2[1 - \Phi(a)] = 2[1 - \Phi(3C_{pm})] \quad (8)$$

$$\mu_y^2 = \mu_T^2 + (\mu_0^2 - \mu_T^2)\exp(-\beta INV) \quad (9)$$

$$\sigma_y^2 = \sigma_T^2 + (\sigma_0^2 - \sigma_T^2)\exp(-\alpha INV) \quad (10)$$

$$P(D \le d_0) = \sum_{d=0}^{d_0} \frac{e^{-np}(np)^d}{d!} \quad (11)$$

$$E(D|D > d_0) \approx \frac{np[1 - \sum_{d=0}^{d_0-1} \frac{e^{-np}(np)^d}{d!}]}{1 - \sum_{d=0}^{d_0} \frac{e^{-np}(np)^d}{d!}} \quad (12)$$

$$R_L = R_I \cdot d_{rl} = R_I[E(D|D > d_0) + (N - n)p] \quad (13)$$

$$C_{pm} = \frac{a}{3} \quad (14)$$

The objective of the modified model in Eqs. (7)-(14) is to determine the optimal quality investment and the C_{pm} value with corresponding improved process mean, improved standard deviation and coefficient of specification limits based on maximization of the expected profit per item. It can't be proven that the modified model in Eqs. (7)-(14) is concave with respect to the combination (INV,C_{pm}) because of the complexity of the cumulative distribution function of the standard normal random variable, $\Phi(\cdot)$, in the objective function, and as a result, the closed-form solution of combination (INV,C_{pm}) might not exist. However, the optimal combination (INV^*, C_{pm}^*) could be found numerically.

The solution procedure for the modified model in Eqs. (7)-(14) is as follows:

Step 1. Give the maximum specified value of C_{pm}.

Step 2. Give the maximum value of *INV*, denoted by INV_{max} .

Step 3. For a given value of *INV*, the corresponding values of μ_y and σ_y may be obtained from Eqs. (3)-(4).

Step 4. Compute the $ETP(INV, C_{pm})$ for all of the combination (*INV*, C_{pm})

Step 5. Let $INV = INV + 0.01$. Repeat Steps 3-4 until $INV = INV_{max}$.

Step 6. Let $C_{pm} = C_{pm} + 0.01$. Repeat Steps 2-5 until the maximum specified value of C_{pm}.

Step 7. By adopting the aforementioned direct search approach, the $ETP(INV, C_{pm})$ with the maximum value is the optimal solution.

The C_{pm}or C_{pmk} index is used for pursuing the minimum bias and variability of product. Hence, the high value of C_{pm}or C_{pmk} can assure the probability of obtaining the target value of output product. The maximum value of C_{pm} can be set at 5/3 because this value indicates that the process is excellent. In addition, from Eq. (7), the maximum value of *INV* may be set at $A_2 - \frac{INV}{N} \ge 0$; that is, $INV \le A_2 N$. The maximum *EPT* (*INV*, C_{pm}) can be obtained in the given limits of C_{pm} and *INV* because of the finite *INV* with positive expected profit per unit.

3.3 Modified Pulak and Al-Sultan's Models with Quality Investment and Specified Process Capability C_{pmk} Value

Denote L and U as the lower and upper specification limits, respectively. That is, $U = \mu_y + b\sigma_y$ and $L = \mu_y - a\sigma_y$, where $a > 0$ and $b > 0$. Assume the target value of process characteristic, T, is equal to the specification center, i.e., $T = \frac{L+U}{2}$. Then, from the definition of C_{pmk}, it can be noted that

$$C_{pmk} = (1 - \frac{|\mu_y - T|}{\frac{U-L}{2}}) \frac{U-L}{6\sqrt{\sigma_y^2 + (\mu_y - T)^2}} = (1 - \frac{|-(b-a)\sigma_y|}{\frac{(b+a)\sigma_y}{2}}) \frac{(a+b)\sigma_y}{6\sqrt{\sigma_y^2 + [\mu_y - \frac{2\mu_y + (b-a)\sigma_y}{2}]^2}} = \frac{(b+a) - |a-b|}{6\sqrt{1 + \frac{(b-a)^2}{4}}} \quad (15)$$

In a similar way, the modified Pulak and Al-Sultan's model with quality investment and C_{pmk} value can be written as follows:

Maximize

$$ETP(INV, a, b) = [A_1 - \frac{R_L}{N} - I_c - c\mu_y] + [A_2 - A_1 + \frac{R_L}{N} + (1 - \frac{n}{N})I_c]P(D \le d_0) - \frac{INV}{N} \quad (16)$$

where

$$\mu_y^2 = \mu_T^2 + (\mu_0^2 - \mu_T^2)\exp(-\beta INV) \quad (17)$$

$$\sigma_y^2 = \sigma_T^2 + (\sigma_0^2 - \sigma_T^2)\exp(-\alpha INV) \quad (18)$$

$$C_{pmk} = \frac{(b+a) - |a-b|}{6\sqrt{1 + \frac{(b-a)^2}{4}}} \quad (19)$$

$$p = 1 - [\Phi(\frac{U-\mu_y}{\sigma_y}) - \Phi(\frac{L-\mu_y}{\sigma_y})] = 2 - \Phi(b) + \Phi(a) \quad (20)$$

$$P(D \le d_0) = \sum_{d=0}^{d_0} \frac{e^{-np}(np)^d}{d!} \quad (21)$$

$$E(D|D > d_0) \approx \frac{np[1 - \sum_{d=0}^{d_0-1} \frac{e^{-np}(np)^d}{d!}]}{1 - \sum_{d=0}^{d_0} \frac{e^{-np}(np)^d}{d!}} \quad (22)$$

$$R_L = R_I \cdot d_{rl} = R_I[E(D|D > d_0) + (N - n)p] \quad (23)$$

The objective of the modified model in Eqs. (16)-(23) is to determine the optimal quality investment and the coefficients of specification limits with corresponding improved process mean and standard deviation based on maximization of the expected profit per item subject to the constraints. Since it cannot be shown that the modified model in Eqs. (16)-(23) is concave with respect to the combination (INV, a, b) because the cumulative distribution function of the standard normal random variable, $\Phi(\cdot)$, in the objective function is complicated, the closed-form solution of combination (INV, a, b) may not be available. However, the optimal combination (INV^*, a^*, b^*) could be determined numerically.

The solution procedure for the modified model in Eqs. (12)-(16) is presented as follows:

Step 1. Give the maximu

Step 2. m specified value of C_{pmk}.
Step 3. Adopt method developed by Chen and Tsai (2003) to determine all of the combinations (a, b) which satisfy the maximum specified value of C_{pmk}.
Step 4. Give the maximum value of *INV*, denoted by INV_{max} .
Step 5. For a given value of *INV*, the corresponding values of μ_y and σ_y can be obtained from Eqs. (17)-(18).
Step 6. Compute the $ETP(INV, a, b)$ for all of the combination (*INV*, *a*, *b*)
Step 7. Let $INV = INV + 0.01$. Repeat Steps 4-5 until $INV = INV_{max}$.
Step 8. By applying the above-mentioned direct search method, the $ETP(INV, a, b)$ with the maximum value is the optimal solution.

The maximum value of C_{pmk} can be set at 5/3 because this value shows that the process is in good quality. Also, from Eq. (16), the maximum value of *INV* can be set at $A_2 - \frac{INV}{N} \geq 0$; i.e., $INV \leq A_2 N$. The maximum $EPT(INV, a, b)$ can be obtained in the given limits of C_{pmk} and *INV* because of the finite *INV* with positive expected profit per unit.

4. Numerical Example and Sensitivity Analysis

4.1 Numerical Example 1

Consider the filling process example provided in Pulak and Al-Sultan (1996). Some of the model parameters are described as follows: The process characteristic of product from historical data is normally distributed with known process mean $\mu_0 = 11.19$ and standard deviation $\sigma_0 = 1$. The lot size of production is $N = 500$ units. The selling price per unit for the accepted lot is $A_2 = 67.5$ and the selling price per unit for the rejected lot is $A_1 = 80$. The processing cost per unit is $c = 5$, the inspection cost per unit is $I_c = 1$, and the replacement cost per unit for a defective unit is $R_I = 30.5$. The parameters related to the single sampling rectifying inspection plan are: the sample size $n = 36$ and the acceptance number $d_0 = 0$. The maximum specified value of the process capability index is $C_{pm} = 2.0$ and the maximum value of the quality investment is $INV = 200$.

With application of quality investment policy, the function coefficients β and α for the process mean and standard deviation can be estimated through the regression analysis, and their estimated values are respectively assumed to be 0.1 and 0.5. Moreover, assume that the target value of the process mean $\mu_T = 12.5$ and the target value of the standard deviation $\sigma_T = 0$. Although "the standard deviation of process characteristic is zero" never occurs practically, this assumed value is actually the target value of process standard deviation for long-term quality improvement. Solving Eqs. (7)-(14) for the modified model leads to that $INV = 0$ and $C_{pm} = 0.6$ with corresponding $a = 1.8$ (i.e., $L = 9.39$ and $U = 12.99$) and $ETP(INV, C_{pm}) = 20.141$.

Table 1 lists some combinations of quality investment and specified value of C_{pm} for examining their effects on the expected profit per unit. For a given value of C_{pm}, the maximum expected profit per unit may be obtained. Based on Table 1, the numerical results indicate that the expected profit per unit is a concave function for various values of C_{pm}.

Table 2 present the sensitivity analysis of some model parameters for this example. From Table 2, the following observations may be drawn:

1. As the lot size increases, the quality investment is not influenced, the length of specification limits of process characteristic is not influenced, and the expected profit per unit increases.
2. As the known standard deviation of process characteristic increases, the quality investment, the length of specification limits of process characteristic, and the expected profit per unit are not influenced.
3. As the inspection cost per unit increases, the quality investment is not influenced, the length of specification limits of process characteristic is not influenced, and the expected profit per unit decreases.
4. As the selling price per unit for the rejected lot increases, the quality investment is not influenced, the length of specification limits of process characteristic decreases, and the expected profit per unit increases.
5. As the selling price per unit for the accepted lot increases, the quality investment is not influenced, the length of specification limits of process characteristic increases, and the expected profit per unit increases.
6. As the processing cost per unit increase, the quality investment is not influenced, the length of specification limits of process characteristic the specification limits of product is also not influenced, and the expected profit per unit decreases.
7. As the cost of replacing a defective unit by an acceptable unit increases, the quality investment is not influenced, the length of specification limits of process characteristic increases, and the expected profit per unit decreases.
8. As the target value of process mean increases, the quality investment decreases, the length of specification limits of process characteristic is not influenced, and the expected profit per unit decreases.
9. As the known process mean increases, the quality investment is not influenced, the length of

specification limits of process characteristic is not influenced, and the expected profit per unit decreases.

10. As the target value of the standard deviation of process characteristic increases, all of the quality investment, the length of specification limits of process characteristic, and the expected profit per unit are not influenced.
11. The function parameter for the process mean increases, the quality investment is not influenced, the length of specification limits of process characteristic is not influenced, and the expected profit per unit increases.
12. The function parameter for the process standard deviation increases, the quality investment is not influenced, the length of specification limits of process characteristic is not influenced, and the expected profit per unit increases.
13. The process capability index value increases, the quality investment is not influenced, the length of specification limits of process characteristic is not influenced, and the expected profit per unit increases.

Table 1: Some Results for the Given Value of C_{pm} in Example 1

C_{pm}	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
a	0.45	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7
INV	55.03	56.18	57.80	0	0	0	0	0	0
$ETP(INV, C_{pm})$	5.175	9.374	13.493	16.030	18.912	20.141	19.044	16.398	13.981
C_{pm}	1.0	1.02	1.04	1.06	1.08	1.10	1.12	1.14	1.16
a	3	3.06	3.08	3.18	3.24	3.30	3.36	3.42	3.48
INV	58.27	58.33	58.40	58.48	0	0	0	0	0
$ETP(INV, C_{pm})$	12.904	12.723	12.482	12.170	11.964	11.873	11.798	11.736	11.685
C_{pm}	1.18	1.20	1.22	1.24	1.26	1.28	1.30	1.32	1.34
a	3.54	3.60	3.66	3.72	3.78	3.84	3.90	3.96	4.02
INV	0	0	0	0	0	0	0	0	0
$ETP(INV, C_{pm})$	11.644	11.610	11.583	11.561	11.543	11.529	11.518	11.509	11.502
C_{pm}	1.50	1.67	1.8	1.9	2.0				
a	4.50	5.01	5.4	5.7	6.0				
INV	0	0	60.65	60.81	61.01				
$ETP(INV, C_{pm})$	11.481	11.478	4.924	4.878	4.845				

Table 2: Sensitivity Analysis for Some Model Parameters in Example 1

N	$(INV, C_{pm}, ETP(INV, C_{pm}))$
250	(0, 0.60, 20.123)
400	(0, 0.60, 20.136)
500	(0, 0.60, 20.141)
600	(0, 0.60, 20.143)
750	(0, 0.60, 20.147)
σ_0	$(INV, C_{pm}, ETP(INV, C_{pm}))$
0.5	(0, 0.60, 20.141)
0.8	(0, 0.60, 20.141)
1.0	(0, 0.60, 20.141)
1.2	(0, 0.60, 20.141)
1.5	(0, 0.60, 20.141)
I_c	$(INV, C_{pm}, ETP(INV, C_{pm}))$
0.5	(0, 0.60, 20.606)
0.8	(0, 0.60, 20.327)
1.0	(0, 0.60, 20.141)
1.2	(0, 0.60, 19.957)
1.5	(0, 0.60, 19.681)
A_1	$(INV, C_{pm}, ETP(INV, C_{pm}))$
70	(0, 0.77, 11.951)
80	(0, 0.60, 20.141)
96	(0, 0.56, 35.337)
120	(0, 0.53, 58.775)
A_2	$(INV, C_{pm}, ETP(INV, C_{pm}))$
33.75	(0, 0.53, 18.887)
54	(0, 0.56, 19.425)
67.5	(0, 0.60, 20.141)
70	(0, 0.62, 20.374)
c	$(INV, C_{pm}, ETP(INV, C_{pm}))$
2.5	(0, 0.6, 48.116)
4	(0, 0.6, 31.331)
5	(0, 0.6, 20.141)
6	(0, 0.6, 8.950)
R_I	$(INV, C_{pm}, ETP(INV, C_{pm}))$
15.25	(0, 0.57, 21.273)
24.4	(0, 0.59, 20.557)
30.5	(0, 0.60, 20.141)
36.6	(0, 0.62, 19.768)
45.75	(0, 0.63, 19.267)
μ_T	$(INV, C_{pm}, ETP(INV, C_{pm}))$
10	(57.84, -, 25.958)
11	(40.25, 0.6, 20.996)
12.5	(0, 0.6, 20.141)
13	(0, 0.6, 20.141)
14	(0, 0.6, 20.141)
μ_0	$(INV, C_{pm}, ETP(INV, C_{pm}))$
10	(0, 0.6, 26.091)
11	(0, 0.6, 21.091)
11.19	(0, 0.6, 20.141)
12	(0, 0.6, 16.091)
σ_T	$(INV, C_{pm}, ETP(INV, C_{pm}))$
0	(0, 0.6, 20.141)
0.2	(0, 0.6, 20.141)
0.4	(0, 0.6, 20.141)
0.6	(0, 0.6, 20.141)
α	$(INV, C_{pm}, ETP(INV, C_{pm}))$
0.1	(0, 0.60, 20.141)
0.2	(0, 0.60, 20.141)
0.3	(0, 0.60, 20.141)
0.4	(0, 0.60, 20.141)

0.5	(0, 0.60, 20.141)
β	$(INV, C_{pm}, ETP(INV, C_{pm}))$
0.1	(0, 0.60, 20.141)
0.2	(0, 0.60, 20.141)
0.3	(0, 0.60, 20.141)
0.4	(0, 0.60, 20.141)
0.5	(0, 0.60, 20.141)
C_{pm}	$(INV, C_{pm}, ETP(INV, C_{pm}))$
0.8	(0, 0.60, 20.141)
0.9	(0, 0.60, 20.141)
1.0	(0, 0.60, 20.141)
1.1	(0, 0.60, 20.141)
1.2	(0, 0.60, 20.141)

4.2 Numerical Example 2

In this example, most numerical values are set identical to those in Example 1 except for $C_{pmk} = k_m = 1$. Solving Eqs. (16)-(23) for the second modified model results in that $INV = 58.27$ with corresponding $\mu_y = 12.5, \sigma_y \to 0, a = 3, b = 3$, and $ETP(INV, a, b) = 12.904$, which indicates that (1) the optimal specification limits should be set as the symmetric tolerance; (2) the process improvement through quality investment is obviously able to assure the output product quality. Table 3 presents the sensitivity analysis of some model parameters for Example 2. From Table 3, it may be shown that (1) if the quality investment is involved in the process improvement, then the improved process parameters (i.e., process mean and standard deviation) can attain the specified value of the process capability index C_{pmk}; (2) the known process standard deviation, the known process mean, the target value of process mean and the target value of process standard deviation significantly affect the quality investment; (3) if the process does not consider the quality investment, the value of the process capability index C_{pmk} could be satisfied for assuring the output product quality by setting the symmetric specification limit coefficient.

5. Conclusions

In the present paper, two modified Pulak and Al-Sultan's models are developed by including quality investment and the specified value of the process capability index C_{pm} or C_{pmk}. The decision variables in the two modified models, e.g., the coefficients of specification limits and quality investment, are determined by maximizing the expected profit per item. The direct search method is applied to determine the optimal solution for the two modified models. From the two presented numerical examples and their sensitivity analyses, we may have the following conclusions: (1) the target value of process mean has a major effect on quality investment; (2) the combination of parameters for sampling inspection plan, the unit selling price for the accepted lot, the unit selling price for the rejected lot, the processing cost per unit, the target value of process mean and the known process mean may significantly influence the expected profit per unit; (3) if the process improvement includes the quality investment, the improved process mean and standard deviation are able to attain the specified value of the process capability index C_{pmk}; (4) if the quality investment is not involved in the process improvement, the value of the process capability index needs to be satisfied for assuring the output product quality by setting the symmetric specification limit coefficient.

The management implication of two modified models is that the manufacturer should provide the specification limits of process characteristic and quality investment to improve the product quality under the constrained loss of customers, which can definitely promote the supply chain's profit, including the supplier, manufacturer, retailer and customer. The integrated application of this study can be available for production of product, process control and improvement, and quality assurance of product to the customer. The satisfaction level will have the significant promotion in the buyer-seller system.

For the manufacturing industry, the process characteristic of product is skewed or follows a non-normal distribution may adopt the asymmetric tolerances model for the transformed data. The extension of the present work may consider Taguchi's quality loss function for measuring the cost of conforming products in the supply chain system with quality assurance and will be left to further study.

Table 3: Sensitivity Analysis for Some Model Parameters in Example 2

N	$(INV, a, b, ETP(INV, a, b))$
250	(58.27, -, -, 12.752)
400	(58.27, -, -, 12.867)
500	(58.27, -, -, 12.904)
600	(58.27, -, -, 12.930)
750	(58.27, -, -, 12.955)
σ_0	$(INV, a, b, ETP(INV, a, b))$
0.5	(55.49, -, -, 12.935)
0.8	(57.35, -, -, 12.982)
1.0	(58.27, -, -, 12.904)
1.2	(58.98, -, -, 12.955)
1.5	(59.90, -, -, 12.876)
I_c	$(INV, a, b, ETP(INV, a, b))$
0.5	(58.27, -, -, 13.306)
0.8	(58.27, -, -, 13.065)
1.0	(58.27, -, -, 12.904)
1.2	(58.27, -, -, 12.743)
1.5	(58.27, -, -, 12.502)
A_1	$(INV, a, b, ETP(INV, a, b))$
70	(0, 3, 3, 11.609)
80	(58.27, -, -, 12.904)
96	(58.27, -, -, 25.537)

120	(58.27, -, -, 44.487)
A_2	$(INV, a, b, ETP(INV, a, b))$
33.75	(58.27, -, -, 5.802)
54	(58.27, -, -, 10.063)
67.5	(58.27, -, -, 12.904)
70	(58.27, -, -, 14.800)
c	$(INV, a, b, ETP(INV, a, b))$
2.5	(58.27, -, -, 44.145)
4	(58.27, -, -, 25.400)
5	(58.27, -, -, 12.904)
6	(0, 3, 3, 1.341)
R_I	$(INV, a, b, ETP(INV, a, b))$
15.25	(58.27, -, -, 13.435)
24.4	(58.27, -, -, 13.117)
30.5	(58.27, -, -, 12.904)
36.6	(58.27, -, -, 12.691)
45.75	(58.27, -, -, 12.524)
μ_T	$(INV, a, b, ETP(INV, a, b))$
10	(58.27, -, -, 24.100)
11	(58.27, -, -, 19.110)
12.5	(58.27, -, -, 12.904)
13	(0, 3, 3, 12.515)
14	(0, 3, 3, 12.515)
μ_0	$(INV, a, b, ETP(INV, a, b))$
10	(0, 3, 3, 18.481)
11	(0, 3, 3, 13.481)
11.19	(58.27, -, -, 12.904)
12	(58.27, -, -, 12.893)
12.5	(58.24, -, -, 12.967)
13	(58.29, -, -, 12.821)
σ_T	$(INV, a, b, ETP(INV, a, b))$
0	(58.27, -, -, 12.904)
0.2	(0, 3, 3, 12.515)
0.4	(0, 3, 3, 12.515)
0.6	(0, 3, 3, 12.515)
0.8	(0, 3, 3, 12.515)
1.0	(0, 3, 3, 12.515)
α	$(INV, a, b, ETP(INV, a, b))$
0.1	(0, 3, 3, 12.515)
0.2	(0, 3, 3, 12.515)
0.3	(0, 3, 3, 12.515)
0.4	(0, 3, 3, 12.515)
0.5	(58.27, -, -, 12.904)
β	$(INV, a, b, ETP(INV, a, b))$
0.1	(58.27, -, -, 12.904)
0.2	(0, 3, 3, 12.515)
0.3	(0, 3, 3, 12.515)
0.4	(0, 3, 3, 12.515)
0.5	(0, 3, 3, 12.515)
C_{pmk}	$(INV, a, b, ETP(INV, a, b))$
0.8	(0, 2.4, 2.4, 16.368)
0.9	(0, 2.7, 2.7, 13.968)
1.0	(58.27, -, -, 12.904)
1.1	(0, 3.3, 3.3, 11.870)
1.2	(0, 3.6, 3.6, 11.609)

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